Math 115

Spring 2019

Lecture 21

$? \; a^2 + b^2 = c^2 \; ?$

$y = mx + b \quad ? \quad d = rt$

Factor out the GCF:

1) $36x^3 + 24x = 12x(3x^2 + 2)$

2) $8x^3y^2 - 24x^2y^3 = 8x^2y^2(x - 3y)$

3) $3x(2x - 5y) + 4y(2x - 5y)$

$= (2x - 5y)(3x + 4y)$

Factor by grouping:

1) $x^2 + 2x + xy + 2y$

$= x(x+2) + y(x+2) = \boxed{(x+2)(x+y)}$

2) $x^2 - 6x + xy - 6y$

$= x(x-6) + y(x-6) = \boxed{(x-6)(x+y)}$

3) $3x^3 + 2y^3 + 3xy^2 + 2x^2y$

$= 3x(x^2 + y^2) + 2y(y^2 + x^2) = \boxed{(x^2+y^2)(3x+2y)}$

Find the remaing factor:

1) $x^2 + 11x + 30 = (x+6)(x+5)$

2) $x^2 - 14x + 24 = (x-2)(x-12)$

3) $x^2 - 7x - 18 = (x+2)(x-9)$

Factor Completely:

1) $x^2 + 9x + 20 = (x+4)(x+5)$

LC=1

1,20
2,10
4,5 ✓

2) $x^2 - 13x + 36 = (x-4)(x-9)$

LC=1

1,36
2,18
3,12
4,9 ✓
6,6

3) $x^2 + 3x - 39$

LC=1

$= (x + \quad)(x - \quad)$

-1,39
-3,13

Prime

1,25
5,5

4) $x^2 - 10x - 25$

LC=1

$= (x + \quad)(x - \quad)$

Prime

Factor Completely:

1) $2x^2 + 7x + 6 = 2x^2 + 3x + 4x + 6$

P=12
S=7
12

$= x(2x+3) + 2(2x+3)$

$= (2x+3)(x+2)$

1,12
2,6
3,4

2) $8x^2 - 14x + 3$

P=24
S=-14
24

$= 8x^2 - 2x - 12x + 3$

$= 2x(4x-1) - 3(4x-1)$

$= (4x-1)(2x-3)$

-1,-24
-2,-12
-3,-8
-4,-6

3) $10x^2 - 11x - 6$

P=-60
S=-11
-60

$= 10x^2 + 4x - 15x - 6$

$= 2x(5x+2) - 3(5x+2)$

$= (5x+2)(2x-3)$

1,-60
2,-30
3,-20
4,-15
5,-12
6,-10

Use Special Factoring of binomials to factor

1) $x^2 - 25 = x^2 - 5^2$

$A^2 - B^2 = (x+5)(x-5)$

2) $x^2 + 9 = x^2 + 3^2$

$= A^2 + B^2$

Prime

3) $x^3 - 8$

$= x^3 - 2^3$

$A^3 - B^3$

$= (x-2)(x^2+2x+4)$

4) $x^3 + 64$

$= x^3 + 4^3$

$A^3 + B^3$

$= (x+4)(x^2-4x+16)$

Factor Completely:

1) $x^2 + 12x + 36$

P=36

S=12

36

1,36
2,18
3,12
4,9
6,6

$= (x+6)(x+6)$

$= (x+6)^2$

2) $x^2 - 16x + 64$

P=64

S=-16

$= (x-8)(x-8)$

$= (x-8)^2$

1,64
2,32
4,16
8,8

3) $4x^2 + 20x + 25 = 4x^2 + 10x + 10x + 25$

P=100

S=20

100

1,100
2,50
4,25
5,20
✓10,10

$= 2x(2x+5) + 5(2x+5)$

$= (2x+5)(2x+5)$

$= (2x+5)^2$

Factor Completely:

1) $16x + 20$

$= \boxed{4(4x+5)}$

2) $x^2 - 81 = x^2 - 9^2$

$= \boxed{(x+9)(x-9)}$

3) $8x^2 + 23x - 3$

$P = -24$, $S = 23$; $-1, 24$; -24

$= 8x^2 + 24x - 1x - 3$

$= 8x(x+3) - 1(x+3)$

$= \boxed{(x+3)(8x-1)}$

4) $8x^3 + 27$

$= (2x)^3 + (3)^3$

$A^3 + B^3$

$= \boxed{(2x+3)(4x^2 - 6x + 9)}$

Factor Completely:

1) $x^3 - 6x^2 + 7x - 42$

$= x^2(x-6) + 7(x-6) = \boxed{(x-6)(x^2+7)}$

2) $3x^2 + 4x - 4 = 3x^2 - 2x + 6x - 4$

$P = -12$, $S = 4$; -2 & 6; -12

$= x(3x-2) + 2(3x-2)$

$= \boxed{(3x-2)(x+2)}$

3) $x^3 + 64y^3$

$= (x)^3 + (4y)^3$

$= \boxed{(x+4y)(x^2 - 4xy + 16y^2)}$

4) $36x^2 - 12x + 1$

$P = 36$, $S = -12$; -6 & -6; 36

$= 36x^2 - 6x - 6x + 1$

$= 6x(6x-1) - 1(6x-1)$

$= (6x-1)(6x-1)$

$= \boxed{(6x-1)^2}$

Factor Completely:

1) $12x^2 - 47x - 4 = 12x^2 - 48x + x - 4$

$P = -48$

$S = -47$

$1 \& -48$

-48

$= 12x(x-4) + 1(x-4)$

$= \boxed{(x-4)(12x+1)}$

2) $8x^2 + 23x - 3$

$P = -24$

$S = 23$

$-1 \& 24$

-24

$= 8x^2 + 24x - x - 3$

$= 8x(x+3) - 1(x+3)$

$= \boxed{(x+3)(8x-1)}$

3) $x^2 + 4$

$= \boxed{\text{Prime}}$

4) $x^4 - 16$

$= (x^2)^2 - (4)^2$

$A^2 - B^2$

$= (x^2+4)(x^2-4)$

$= \boxed{(x^2+4)(x+2)(x-2)}$

Zero-Factor Property or

Zero-Product Rule

If $A \cdot B = 0$, then $A = 0$ or $B = 0$

maybe both 0.

Solve $(x-1)(x+8) = 0$

by Zero-Product rule

$x - 1 = 0$ or $x + 8 = 0$

$\boxed{x = 1}$ $\boxed{x = -8}$

$\{-8, 1\}$

Solve $(2x-5)(3x+7)=0$

By Zero-Factor Property

$2x-5=0$ OR $3x+7=0$

$2x=5$ $3x=-7$

$\boxed{x=\frac{5}{2}}$ $\boxed{x=\frac{-7}{3}}$

$\left\{\frac{-7}{3}, \frac{5}{2}\right\}$

Solving Polynomial equations

① Make one side equal to zero.

② Factor the other side completely.

③ use Zero-Factor Property to proceed.

Solve $x^2+x=12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x+4=0$ or $x-3=0$

$\boxed{x=-4}$ $\boxed{x=3}$

$\{-4, 3\}$

Solve $x^2 - 24 = 2x$

$x^2 - 24 - 2x = 0$

$x^2 - 2x - 24 = 0$

$(x + 4)(x - 6) = 0$

By Z.F.P.

$x + 4 = 0$ $\quad$ $x - 6 = 0$

$x = -4$ $\quad$ $x = 6$

$\{-4, 6\}$

Solve $(x + 5)(x - 1) = 16$

Hint: FOIL, Simplify, Then proceed.

$x^2 - x + 5x - 5 = 16$

$x^2 + 4x - 5 - 16 = 0$

$x^2 + 4x - 21 = 0$

$(x - 3)(x + 7) = 0$

By Z.F.P.

$x - 3 = 0$ or $x + 7 = 0$

$x = 3$ $\quad$ $x = -7$

$\{-7, 3\}$

Solve $(2x+1)(x+2)=20$

Hint: See last example

$2x^2+4x+x+2-20=0$

$2x^2+5x-18=0$

$P=-36$

$S=5$

-4 & 9

-36

$2x^2-4x+9x-18=0$

$=2x(x-2)+9(x-2)=0$

$=(x-2)(2x+9)=0$

By Z.F.P.

$x-2=0$ $\quad$ $2x+9=0$

$\boxed{x=2}$ $\quad$ $\boxed{x=\frac{-9}{2}}$

$\left\{\frac{-9}{2}, 2\right\}$

Due wednesday: SG 15

Project III: Factoring Polynomial

Due Monday

Lisa has \$3.80 in nickels & Quarters. Total # of coins is 20. How many of each?

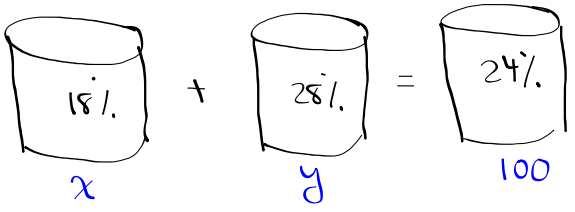
$\div 5\begin{cases} N+Q=20 \\ 5N+25Q=380 \end{cases} \xrightarrow{-1} \begin{cases} N+Q=20 \\ N+5Q=76 \end{cases}$

$4Q=56$

$\boxed{Q=14}$

14 Quarters & 6 Nickels

John needs 100 liters of 24% alcohol soln. He has access to unlimited supply of 18% & 28% alcohol soln. How many liters of each?



$$\begin{cases} x + y = 100 \\ .18x + .28y = .24(100) \end{cases}$$

$$\div 2 \begin{cases} x + y = 100 \\ 18x + 28y = 2400 \end{cases} \Rightarrow \begin{matrix} -9 \\ \end{matrix} \begin{cases} x + y = 100 \\ 9x + 14y = 1200 \end{cases}$$

$$5y = 300 \Rightarrow y = 60$$

60 L of 28% Alcohol

&

40 L of 18% Alcohol

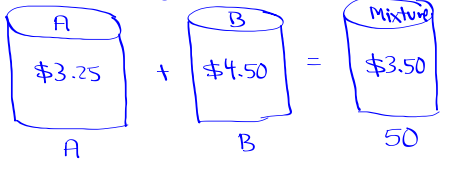
Store has two brands of coffee:

Brand A is \$3.25/lb.

Brand B is \$4.50/lb.

The manager needs 50 lb @ \$3.50/lb.

How many pounds of each?



$$\begin{cases} A + B = 50 \\ 3.25A + 4.50B = 3.50(50) \end{cases}$$

$$100 \begin{cases} A + B = 50 \\ 3.25A + 4.50B = 3.50(50) \end{cases} \div 5 \begin{cases} A + B = 50 \\ 325A + 450B = 350(50) \end{cases}$$

$$\div 5 \begin{cases} A + B = 50 \\ 65A + 90B = 3500 \end{cases} \Rightarrow \begin{matrix} -13 \\ \end{matrix} \begin{cases} A + B = 50 \\ 13A + 18B = 700 \end{cases}$$

$$5B = 50 \Rightarrow B = 10$$

10 lb. from Brand B

&

40 lb. from Brand A

Jose invested $5000 for one year in two simple interest accounts. one paid 4% & other paid 6% interest. He earned $256 in total interest. How much per account?

$x @ 4%.
$y @ 6%.

$$\begin{cases} x + y = 5000 \\ .04x + .06y = 256 \end{cases} \quad \times 100$$

$$\begin{cases} x + y = 5000 \\ 4x + 6y = 25600 \end{cases} \quad \div 2$$

$$\Rightarrow \begin{cases} x + y = 5000 \\ 2x + 3y = 12800 \end{cases} \quad -2($$

$y = 2800$

$2800 @ 6%.
$2200 @ 4%.